A Maximum Likelihood Method for Fitting CMDs

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Abstract We have developed a method for fitting colour magnitude diagrams of clusters and associations. Its advantages over other techniques are (1) the inclusion of binaries (2) robust parameter uncertainties and (3) a goodness of fit statistic. The technique was described in Jeffries & Naylor (2006; MNRAS 373 1251), but we have made significant improvements since, which are included in the code which is available at http://www.astro.ex.ac.uk/people/timn/tau-squared.

WHAT'S THE PROBLEM? Despite the acquisition of many excellent colour-magnitude diagrams for clusters and associations, and the calculation of good models, fitting these models to the data is still largely done "by eye". There is a good reason for this, even though it is not clearly elucidated in the literature. Were the data drawn from a single star sequence the problem would become one of fitting an arbitrary line to data points with uncertainties in two dimensions. Even this is not a straightforward problem, but was largely solved by Flannery & Johnson (1982; ApJ 263 166). However, this solution remains little used for the obvious reason that our data are not drawn from a single-star sequence, but from a population which contains a large fraction of binary stars. Faced with this, most galactic astronomers have taken the "by eye" approach, fitting the single-star model sequences to the lower envelope of the data. In contrast we have developed the following statistically rigorous solution.

OUTLINE SOLUTION

1) Simulate a million stars with the parameters of interest (e.g. distance modulus and age), and bin them in colour-magnitude space to create the "grey scale" model in Figure 1.
2) Assuming the uncertainties for the data points are small*, evaluate the model at the position of each data point i, to obtain P_i for each data point.
3) Multiplying these values together gives a goodness-of-fit parameter, though we actually use $-2\ln\prod P_i = \tau^2$, since this is related to χ^2.
4) Change the parameters until you find the best (lowest) value of τ^2.
5) There are then techniques for finding probability that this is a good fit ($\Pr(\tau^2)$).
6) If the model is a good fit, one can then determine uncertainties in the parameters in a similar way to χ^2.

*If the uncertainties are large, convolve the image with the uncertainty for the data point before taking the value.

FORMAL DEFINITION The formal definition of our statistic is given by

$$\tau^2 = -2\ln\prod_{i=1,N} P_i = -2\sum_{i=1,N}\ln\int U_i(c-c_i, m-m_i)\rho(c,m)\,dc\,dm,$$

where P_i, the probability for a single data point at (c_i, m_i) is given by the integral of the model ρ multiplied by U_i the probability distribution due to the uncertainties for that data point. It can be formally derived from maximum likelihood theory, and as such can be viewed as either a Bayesian or perfectly respectable Frequentist method.

We can show that if the model is a single sequence with uncertainties in one dimension τ^2 is identical to χ^2, i.e. χ^2 is a special case of τ^2. If ρ is a line and U_i a one-dimensional Gaussian. Then the product is only non-zero where the two intersect, and has a value proportional to the value of the Gaussian at that point. Thus the integral reduces to $\exp((c-c_i)^2/\sigma_i^2)$, leading to the normal form for χ^2.

AN INTUITIVE INTERPRETATION Our solution to this problem can be envisaged in the following way. Imagine moving the grey scale in Figure 1 up and down over the data points, and collecting the values of the probability at the position of each data point. The value of the product of all these values is clearly maximised when the data of are placed correctly magnitude, i.e. when the distance is correct. One can derive uncertainties in the fitted parameters in a similar way to a χ^2 analysis, and we show in Figure 2 the τ^2 space for fitting the data of Figure 1, with a confidence contour.

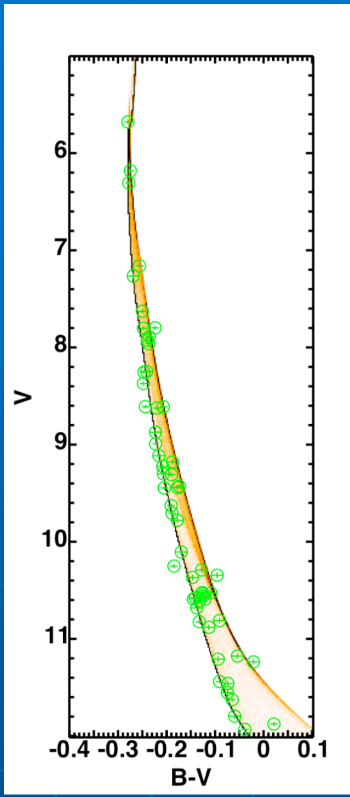

FIGURE 1 The grey scale is the best fit model isochrone to the dereddened members of NGC6530 (circles). The model uses the Padova isochrones and Bessell's bolometric corrections. The age is 5.5Myr and the distance modulus 10.7

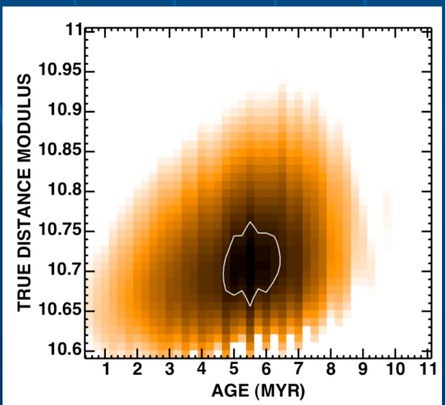

FIGURE 2 A grey scale plot of τ^2 as a function of age and distance modulus for the model and data of Figure 1. The white contour is the 67% confidence level.

PARAMETER UNCERTANTIES One can undertake a grid search to find the best-fitting values for a pair of parameters in an identical fashion to a conventional χ^2 fit. This then produces a grid such as Figure 2. From the definition of τ^2 each point in the grid has an associated probability. The ratio of the sum of the probability for all the pixels below a given τ^2 to the sum of the whole image gives the probability that the correct solution (and hence pair of parameters) lies within pixels of that τ^2 or below. This allows one to draw, say 68 percent confidence contours on the τ^2 space.

RESULTS

NGC2547. We performed the first statistically robust test to show that a pre-main-sequence age (in this case 38 Myr) matches that from Li depletion (Naylor & Jeffries 2006, MNRAS 373 1251).

Distances for young clusters and OB associations. We have determined a set of self consistent distances for young groups by fitting the young main-sequence stars (Mayne & Naylor 2008, MNRAS 386 261). This allows us to revise the age ordering of the clusters, which is important for studies of rotational and disc evolution.

NGC2169. We determined the distance, could then find which PMS model isochrones fitted the data best, and hence determine the age as 9±2Myr (Jeffries et al, 2007 MNRAS 376 580).

γ Vel/Vel OB2. We have shown that the γ Vel association is at the same distance as Vel OB2 (Jeffries et al, 2008 MNRAS submitted).

NGC7149. Another group have used the technique to determine the distance to this cluster (Himali et al, 2008 MNRAS accepted).

CONCLUSIONS τ^2 a very powerful technique for extracting robust parameters with uncertainties from colour-magnitude diagrams. Although our own immediate interest is young clusters and associations, we would be interested in collaborating with groups on older clusters. Furthermore the method is very general, and should have many applications to sparse datasets, and datasets with uncertainties in two (or more) dimensions.